Hydraulic head

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Hydraulic head is a specific measurement of water pressure or total energy per unit weight above a datum. It is usually measured as a water surface elevation, expressed in units of length, but represents the energy at the entrance (or bottom) of a piezometer. In an aquifer, it can be calculated from the depth to water in a piezometric well (a specialized water well), and given information of the piezometer's elevation and screen depth. Hydraulic head can similarly be measured in a column of water using a standpipe piezometer by measuring the height of the water surface in the tube relative to a common datum. In both cases, although the measurement is made at the water surface, the hydraulic head measurement represents the total energy at the entrance (base) of the piezometer. The hydraulic head can be used to determine a *hydraulic gradient* between two or more points.

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"Head" in fluid dynamics

In fluid dynamics, **head** is the difference in elevation between two points in a column of fluid, and the resulting pressure of the fluid at the lower point. Head is normally expressed in units of height (e.g. meters), although some texts (incorrectly) refer to it in units of pressure such as pascals (the SI unit). Head refers to the constant right hand side in the incompressible steady version of Bernoulli's equation.

This is best understood by considering a waterwheel: the head is the vertical distance from the top of the waterwheel to the free surface of the millpond.

More generally, when considering a flow, one says that head is lost if energy is dissipated, often through turbulence; equations such as the Prony equation and the Darcy-Weisbach equation have been used to calculate the head loss due to friction. In the context of steam trains, one talks of a *good head of steam*, referring to the pressure in the boiler.

The **static head** of a pump is the maximum height (pressure) it can deliver. The capacity of the pump can be read from its Q-H curve (flow vs. height).

Head is used to describe the energy in incompressible fluids. Head has units of distance and equals the fluid's energy per unit weight. Head is useful in specifying centrifugal pumps because their pumping characteristics tend to be independent of the fluid's density.

There are four types of head used to calculate the total head in and out of a pump:

- *Static head* is due to gravitational force on a column of fluid.
- *Velocity head* is due to the motion of a fluid (kinetic energy).
- *Resistance head* (or *friction head*) is due to the frictional forces against a fluid's motion.
- *Pressure head* is due to other mechanical forces acting on a fluid.

Components of hydraulic head

The total hydraulic head is composed of *pressure head*, *velocity head* and *elevation head*. The pressure head is the equivalent gauge pressure of a column of water at the base of the piezometer, the velocity head is the kinetic energy from the motion of water, and the elevation head is the relative potential energy in terms of an elevation. This can be expressed as:

$$h = \psi + h_v + z$$

where

- h is the hydraulic head (Length in m or ft),
- h_v is the velocity head, measured by a Pitot tube (Length in m or ft),
- ψ is the pressure head, in terms of the elevation difference of the water column relative to the piezometer bottom (Length in m or ft), and
- z is the elevation at the piezometer bottom (Length in m or ft)

In groundwater
studies, the velocity head is assumed to be zero and is ignored. This is because groundwater moves very slowly, and the kinetic energy loss is very low. Thus, for groundwater studies, hydraulic head can be defined simply as:

$$h = \psi + z$$

In an example with a 400 m deep piezometer, with an elevation of 1000 m, and a depth to water of 100 m: z = 600 m, ψ = 300 m, and h = 900 m.

The pressure head can be expressed as:

$$\psi = \frac{P}{\gamma} = \frac{P}{\rho g}$$

where

- P is the gauge pressure (Force per unit area, often Pa or psi),
- γ is the unit weight of water (Force per unit volume, typically $N \cdot m^{-3}$ or lbf/ft^3),
- ρ is the density of the water (Mass per unit volume, frequently $kg \cdot m^{-3}$), and
- g is the gravitational acceleration (velocity change per unit time, often $m \cdot s^{-2}$)

In faster moving water where the Reynolds number exceeds 10, such as in a river, the velocity head can be calculated using Bernoulli's principles as:

$$h_v = \frac{v^2}{2g}$$

where v is relative velocity of the water (distance per unit time, often $m \cdot s^{-1}$ or $ft \cdot s^{-1}$)

Fresh water head

The pressure head is dependent on the density of water, which can vary depending on both the temperature and chemical composition (salinity, in particular). This means that the hydraulic head calculation is dependent on the density of the water within the piezometer. If one or more hydraulic head measurements are to be compared, they need to be standardized, usually to their *fresh water head*, which can be calculated as:

$$h_{fw} = \psi \frac{\rho}{\rho_{fw}} + z$$

where

h_{fw} is the fresh water head (Length, measured in m or ft), and
ρ_{fw} is the density of fresh water (Mass per unit volume, typically in kg·m^{-3})

Hydraulic gradient

The *hydraulic gradient* is a vector gradient
between two or more hydraulic head measurements over the length of the flow path. It is also called the *darcy slope*, since it determines the quantity of a *darcy flux*, or discharge. A dimensionless hydraulic gradient can be calculated between two piezometers as:

$$i = \frac{dh}{dl} = \frac{h_2 - h_1}{\mathrm{length}}$$

where

i is the hydraulic gradient (dimensionless),
dh is the difference between two hydraulic heads (Length, usually in m or ft), and
dl is the flow path length between the two piezometers (Length, usually in m or ft)

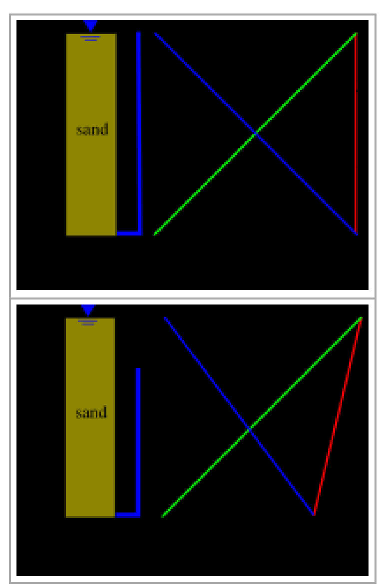
The hydraulic gradient can be expressed in vector notation, using the del operator. This requires a hydraulic head field, which can only be practically obtained from a numerical model, such as MODFLOW. In Cartesian coordinates, this can be expressed as:

$$\nabla h = \left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}, \frac{\partial h}{\partial z}\right) = \frac{\partial h}{\partial x}\mathbf{i} + \frac{\partial h}{\partial y}\mathbf{j} + \frac{\partial h}{\partial z}\mathbf{k}$$

This vector describes the direction of the groundwater flow, where negative values indicate flow along the dimension, and zero indicates *no flow*. As with any other example in physics, energy must flow from high to low, which is why the flow is in the negative gradient. This vector can be used in conjunction with Darcy's law and a tensor of hydraulic conductivity to determine the flux of water in three dimensions.

Hydraulic head in groundwater

The distribution of hydraulic head through an aquifer determines where groundwater will flow. In a hydrostatic



Relation between heads for a *hydrostatic* case and a *downward flow* case.

example (first figure), where the hydraulic head is constant, there is no flow. However, if there is a difference in hydraulic head from the top to bottom due to draining from the bottom (second figure), the water will flow downward, due to the difference in head, also called the *hydraulic gradient*.

Atmospheric pressure

Even though it is convention to use gauge pressure
in the calculation of hydraulic head, it is more correct to use total pressure (gauge pressure + atmospheric pressure), since this is truly what drives groundwater flow. Often detailed observations of barometric pressure are not available at each well through time, so this is often disregarded (contributing to large errors at locations where hydraulic gradients are low or the angle between wells is acute.)

The effects of changes in atmospheric pressure
upon water levels observed in wells has been known for many years. The effect is a direct one, an increase in atmospheric pressure is an increase in load on the water in the aquifer, which increases the depth to water (lowers the water level elevation). Pascal first qualitatively observed these effects in the 1600s, and they were more rigorously described by the soil physicist Edgar Buckingham (working for the USDA) using air flow models in 1907.

Analogs to other fields

Hydraulic head is a measure of energy, and has many analogs in physics and chemistry, where the same mathematical principles and rules apply:

- *Hydraulic head* is analogous to:
  - magnetic monopole
  - electric charge
  - heat (i.e., temperature)
  - concentration
- A continuous "field" of hydraulic heads is analogous to:
  - an electric field
  - a magnetic field
- Similar differential operators can be applied to the fields, to find:
  - the gradient, or the direction of flow
  - the divergence of flow
  - the curl, or if the field is rotating

See also

- Bernoulli's principle, from which we derive pressure head and velocity head
- fluid dynamics
- Pressure head
- Darcy's law
- Total Dynamic Head

References

- Bear, J. 1972. *Dynamics of Fluids in Porous Media*, Dover. ISBN 0-486-65675-6.
- for other references which discuss hydraulic head in the context of hydrogeology, see that page's further reading section