

# Darcy's law

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In fluid dynamics, **Darcy's law** is a phenomenologically derived constitutive equation that describes the flow of a fluid through a porous medium. The law was formulated by Henry Darcy based on the results of experiments<sup>[1]</sup> on the flow of water through beds of sand. It also forms the scientific basis of fluid permeability used in the earth sciences.

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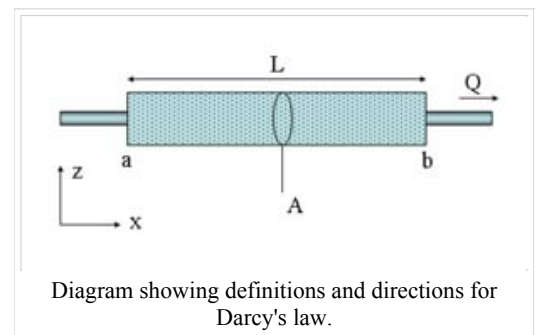
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## Background

Although Darcy's law (an expression of conservation of momentum) was determined experimentally by Darcy, it has since been derived from the Navier-Stokes equations via homogenization. It is analogous to Fourier's law in the field of heat conduction, Ohm's law in the field of electrical networks, or Fick's law in diffusion theory.

One application of Darcy's law is to water flow through an aquifer. Darcy's law along with the equation of conservation of mass are equivalent to the groundwater flow equation, one of the basic relationships of hydrogeology. Darcy's law is also used to describe oil, water, and gas flows through petroleum reservoirs.

## Description



Darcy's law is a simple proportional relationship between the instantaneous discharge rate through a porous medium, the viscosity of the fluid and the pressure drop over a given distance.

$$Q = \frac{-\kappa A}{\mu} \frac{(P_b - P_a)}{L}$$

The total discharge,  $Q$  (units of volume per time, e.g., m<sup>3</sup>/s) is equal to the product of the permeability ( $\kappa$  units of area, e.g. m<sup>2</sup>) of the medium, the cross-sectional area ( $A$ ) to flow, and the pressure drop ( $P_b - P_a$ ), all divided by the dynamic viscosity  $\mu$  (in SI units e.g. kg/(m·s) or Pa·s), and the length  $L$

the pressure drop is taking place over. The negative sign is needed because fluids flow from high pressure to low pressure. So if the change in pressure is negative (in the  $x$ -direction) then the flow will be positive (in the  $x$ -direction). Dividing both sides of the equation by the area and using more general notation leads to

$$q = \frac{-\kappa}{\mu} \nabla P$$

where  $q$  is the flux (discharge per unit area, with units of length per time, m/s) and  $\nabla P$  is the pressure gradient vector. This value of flux, often referred to as the Darcy flux, is not the velocity which the water traveling through the pores is experiencing<sup>[2]</sup>.

The pore velocity ( $v$ ) is related to the Darcy flux ( $q$ ) by the porosity ( $\phi$ ). The flux is divided by porosity to account for the fact that only a fraction of the total formation volume is available for flow. The pore velocity would be the velocity a conservative tracer would experience if carried by the fluid through the formation.

$$v = \frac{q}{\phi}$$

### In 3D

In three dimensions, gravity must be accounted for, as the flow is not affected by the vertical pressure drop caused by gravity when assuming hydrostatic conditions. The solution is to subtract the gravitational pressure drop from the existing pressure drop in order to express the resulting flow,

$$\mathbf{q} = \frac{-\boldsymbol{\kappa}}{\mu} (\nabla P - \rho g \hat{\mathbf{e}}_z)$$

where the flux  $\mathbf{q}$  is now a vector quantity,  $\boldsymbol{\kappa}$  is a tensor of permeability,  $\nabla$  is the gradient operator in 3D,  $g$  is the acceleration due to gravity,  $\hat{\mathbf{e}}_z$  is the unit vector in the vertical direction, pointing downwards and  $\rho$  is the density.

Effects of anisotropy in three dimensions are addressed using a symmetric second-order tensor of permeability:

$$\boldsymbol{\kappa} = \begin{bmatrix} \kappa_{xx} & \kappa_{xy} & \kappa_{xz} \\ \kappa_{yx} & \kappa_{yy} & \kappa_{yz} \\ \kappa_{zx} & \kappa_{zy} & \kappa_{zz} \end{bmatrix}$$

where the magnitudes of permeability in the  $x$ ,  $y$ , and  $z$  component directions are specified. Since this a symmetric matrix, there are *at most* six unique values. If the permeability is isotropic (equal magnitude in all directions), then the diagonal values are equal,

$$\kappa_{xx} = \kappa_{yy} = \kappa_{zz} > 0$$

, while all other components are 0. The permeability tensor can be interpreted through an evaluation the relative magnitudes in each component. For example, rock with highly permeable vertical fractures aligned in the  $x$ -direction will have relatively higher values for  $\kappa_{xx}$  than other component values.

### Assumptions

Darcy's law is a simple mathematical statement which neatly summarizes several familiar properties that groundwater flowing in aquifers exhibits, including:

- if there is no pressure gradient over a distance, no flow occurs (this is hydrostatic conditions),
- if there is a pressure gradient, flow will occur from high pressure towards low pressure (opposite the direction of increasing gradient - hence the negative sign in Darcy's law),
- the greater the pressure gradient (through the same formation material), the greater the discharge rate, and
- the discharge rate of fluid will often be different — through different formation materials (or even through the same material, in a different direction) — even if the same pressure gradient exists in both cases.

A graphical illustration of the use of the steady-state groundwater flow equation (based on Darcy's law and the conservation of mass)

is in the construction of flownets, to quantify the amount of groundwater flowing under a dam.

Darcy's law is only valid for slow, viscous flow; fortunately, most groundwater flow cases fall in this category. Typically any flow with a Reynolds number less than one is clearly laminar, and it would be valid to apply Darcy's law. Experimental tests have shown that for flow regimes with values of Reynolds number up to 10 may still be Darcian. Reynolds number (a dimensionless parameter) for porous media flow is typically expressed as

$$Re = \frac{\rho v d_{30}}{\mu}$$

where  $\rho$  is the density of the fluid (units of mass per volume),  $v$  is the specific discharge (not the pore velocity — with units of length per time),  $d_{30}$  is a representative grain diameter for the porous medium (often taken as the 30% passing size from a grain size analysis using sieves), and  $\mu$  is the dynamic viscosity of the fluid.

## Additional forms of Darcy's law

### Time derivative of flux

For very short time scales, a time derivative of flux may be added to Darcy's law, which results in valid solutions at very small times (in heat transfer, this is called the modified form of Fourier's law),

$$\tau \frac{\partial q}{\partial t} + q = -K \nabla h,$$

where  $\tau$  is a very small time constant which causes this equation to reduce to the normal form of Darcy's law at "normal" times (> nanoseconds). The main reason for doing this is that the regular groundwater flow equation (diffusion equation) leads to singularities at constant head boundaries at very small times. This form is more mathematically rigorous, but leads to a hyperbolic groundwater flow equation, which is more difficult to solve and is only useful at very small times, typically out of the realm of practical use.

### Brinkman term

Another extension to the traditional form of Darcy's law is the Brinkman term, which is used to account for transitional flow between boundaries (introduced by Brinkman in 1947),

$$\beta \nabla^2 q + q = -K \nabla h,$$

where  $\beta$  is an effective viscosity term. This correction term accounts for flow through medium where the grains of the media are porous themselves, but is difficult to use, and is typically neglected.

### Multiphase flow

For multiphase flow, an approximation is to use Darcy's law for each phase, with permeability replaced by phase permeability, which is the permeability of the rock multiplied with relative permeability. This approximation is valid if the interfaces between the fluids remain static, which is not true in general, but it is still a reasonable model under steady-state conditions.

Assuming that the flow of a phase in the presence of another phase can be viewed as single phase flow through a reduced pore network, we can add the subscript  $i$

for each phase to Darcy's law above written for Darcy flux, and obtain for each phase in multiphase flow

$$q_i = -\frac{\kappa_i}{\mu_i} \nabla P_i$$

where  $\kappa_i$  is the *phase permeability* for phase  $i$ . From this we also define relative permeability  $\kappa_{ri}$  for phase  $i$  as

$$\kappa_{ri} = \kappa_i / \kappa$$

where  $\kappa$  is the permeability for the porous medium, as in Darcy's law.

### Forchheimer equation for non-Darcy flow

For a sufficiently high flow velocity, the flow is nonlinear, and Dupuit and Forchheimer has proposed to generalize the flow equation to

$$-\nabla P = \frac{\mu}{\kappa}V + \beta\rho V^2$$

where  $V$  is the flow velocity and  $\beta$  is a factor to be experimentally deduced.

### In membrane operations

In pressure-driven membrane operations, Darcy's law is often used in the form,

$$J = \frac{\Delta P - \Delta \Pi}{\mu(R_f + R_m)}$$

where,

- $J$  is the volumetric flux ( $m.s^{-1}$ ),
- $\Delta P$  is the hydraulic pressure difference between the feed and permeate sides of the membrane ( $Pa$ ),
- $\Delta \Pi$  is the osmotic pressure difference between the feed and permeate sides of the membrane ( $Pa$ ),
- $\mu$  is the dynamic viscosity ( $Pa.s$ ),
- $R_f$  is the fouling resistance ( $m^{-1}$ ), and
- $R_m$  is the membrane resistance ( $m^{-1}$ )

## References

1. ↑ H. Darcy, *Les Fontaines Publiques de la Ville de Dijon*, Dalmont, Paris (1856).
2. ↑ See Stauffer, Philip H. (2006). "Flux Flummoxed: A Proposal for Consistent Usage". *Ground Water* **44** (2): 125–128. doi:10.1111/j.1745-6584.2006.00197.x. for a discussion of the many, sometimes confusing names given to ( $q$ ) in the ground water literature.

## See also

- The darcy unit of fluid permeability
- Hydrogeology
- Groundwater discharge
- Groundwater flow equation
- Groundwater energy balance
- Richards equation
- Darcy friction factor

## External links

Browser-based numerical calculator of permeability using Darcy's law. (<http://www.calctool.org/CALC/eng/fluid/darcy>)

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